MATHEMATICAL MAGIC IN UNDERGRADUATE MATHEMATICAL CLASSES FOR PRE-SERVICE TEACHERS

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Abstract

Nowadays we live in an ever-changing society. The educational context is no exception, and requires a renewal of paradigms. Profound changes to the role and function of the teacher and the students are particularly vital [1], [2], [3]. The current University students have different motivations, different attitudes towards the teaching and learning process, and different responses to specific classroom environments and instructional practices [4]. The challenge that teachers face today is to motivate students to learn, get them to commit to and have an active role in their learning [5].

Considering that it is imperative to make the teaching and learning process of mathematics more stimulating, taking into account modern society and student's interests [6], the authors have developed several strategies to increase the interest and improve the success of students in mathematics. Thus, they decided to use mathematical magic tricks in their classes to raise the motivation of the students, captivating them and stimulating their interest in mathematics. In this work, a case study to evaluate how mathematical magic can contribute to increase students' motivation for learning mathematics is presented. The paper describes how it is conceived as well as the main results.

Keywords: mathematics, mathematics education, recreational mathematics, mathematical magic, higher education.

1 INTRODUCTION

Nowadays, we live in an ever-changing society. The educational context is no exception, and requires a renewal of paradigms [3]. Often, teachers are faced with discouraged students: students with glassy eyes, heads down on desks, students who don´t remember content taught to them the day before! Mathematics classes are no exception. Quite the opposite! One of the problems identified in the failure of mathematics teaching is the lack of motivation that students feel towards the subject [7].

Profound changes to the role and function of both teachers and students are particularly vital. Active, cooperative and participative methodologies of learning that allow a break from magisterial education and the mere transmission of knowledge must be privileged. The greatest challenge that teachers face today is to engage students, motivate them to learn and take an active role in their learning process [4]. According to Matos [5], motivation is an essential factor in any learning since the quality of learning is not only related to the capacity to learn, but also to the level of motivation that we have to carry out this same learning. If students care about mathematics they are far more likely to remember what they learn. If students develop motivation to learn about a subject, they are much more likely to develop long-term understanding [8].

Considering that it is imperative to make the teaching and learning process of mathematics more stimulating, taking into account modern society and student's interests [6], the authors have developed several strategies to increase the interest and improve the success of students in mathematics. Thus, and because they believe that exploring the relationship between magic and mathematics has an enormous potential in developing activities in the classroom to promote the interest in mathematics, they decided to use mathematical magic tricks in their classes to raise the motivation of the students, captivating them and stimulating their interest in mathematics. Performing a magic trick and exploring its mathematical content can be a source of reasoning and mathematical learning which can be empowered by the magic setup. In this work, the authors present a case study to evaluate how mathematical magic can contribute to increase students' motivation for learning mathematics, continuing the preliminary study described in [7].

2 CASE STUDY DESCRIPTION

This case study took place during the classes of the course entitled *Conceitos de Matemática I* (Mathematical Concepts I) in the first semester of the academic year 2018-19. This course is part of the 1st year syllabus of the undergraduate programme "Licenciatura em Educação Básica" which is a three year higher education 1st cycle programme for teacher training (pre-service teachers from kindergarten to 6th grade levels). This course has five hours contact every week, divided into two classes of 2 and 3 hours, during a whole semester (15 weeks).

The topics of this course are numbers and operations. In the first part of the course different numeral systems are addressed and the students learn how to represent (and convert) numbers in different numeral systems and bases. The four basic operations are explored (addition, subtraction, multiplication and division) and their algorithms are detailed not only in the decimal system but also in other bases. The second part of the course is devoted to elementary number theory. Multiples/divisors, divisibility criteria, prime number decomposition and applied problems are addressed. The third and last part of the course is related to decimal numbers, exploring the four operations both with fractions and/or decimal representations. Problem solving is also addressed using different approaches with a strong emphasis in the use of diagrams.

In the beginning of every class some minutes (about 5 to 10) were dedicated to the performance of a mathematical magic trick which, whenever possible, was related to the topic of the class. The explanation of the mathematics underlying the trick was usually presented later during the class or in the next class. In the last case, students were challenged to think about the trick at home. Most of the times, they couldn't find out the explanation by themselves. However, sometimes they brought ideas that were good clues to the full explanation.

In order to illustrate the kind of tricks performed in the classes we describe a selection of three tricks directly related to the topics involved in the classes. The first trick – Guessing a birthday – is related to the representation of numbers in the binary system. The second trick – Guessing a word – is related to the standard addition and subtraction algorithms and constitutes a good example of algebraic reasoning. The third trick – Crystal ball – is related to the concept of multiples and is a good example of a simple algebraic proof.

Guessing a birthday – A volunteer is shown a set of four lists of months and another set of 5 lists of numbers shown in Figure 1. One by one, he is asked if his month (and later day) of birth is, or is not, in the given lists. Based on the answers the magician/teacher "guesses" his date of birth. Without realizing it, the answers given by the volunteer (yes/no) constitute the digits of the number of the month and the day of his birthday in binary code (yes – 1; no – 0).

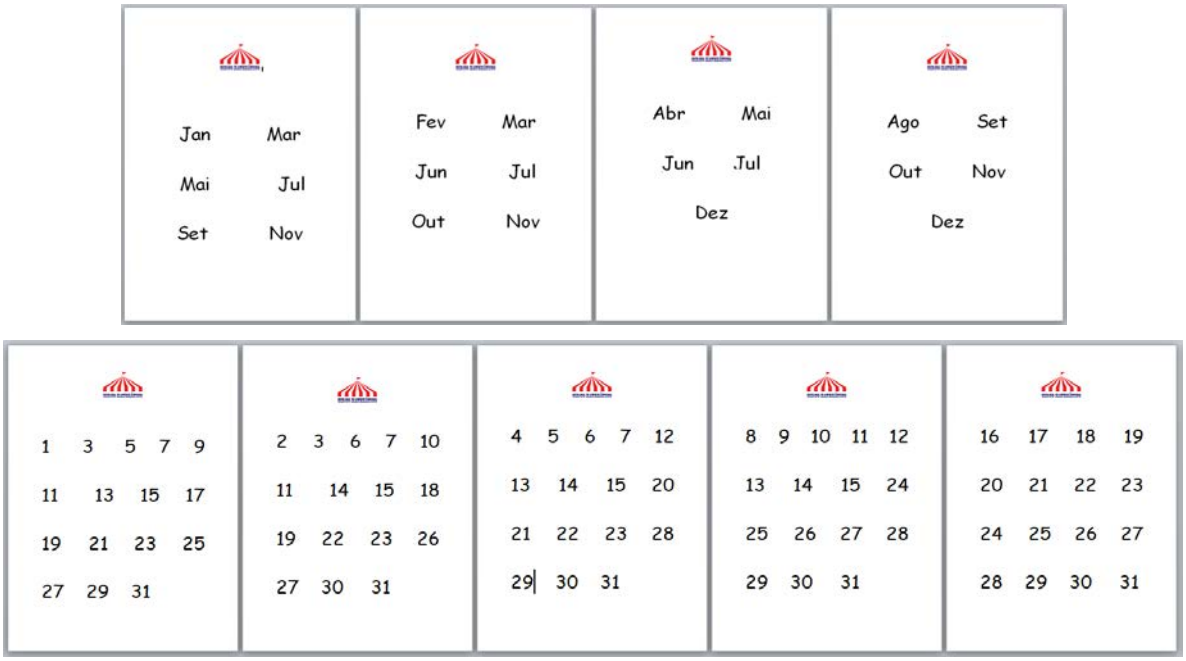

Figure 1. Tables for guessing a birthday.

Guessing a word – The volunteer is asked to guess a word, previously hidden in the room, from a thick book (we used one of the Harry Potter books). In order to guess the word he is asked to find a code. For that purpose he is asked to choose (at random) a three digit number of his choice (digits must be different). He is then asked to reverse the digit order and subtract the smallest number from the largest. He is then asked to reverse the order of the digits of the difference and sum this number to the difference. What he doesn't know is that the final sum is always 1089, irrespectively of the starting number. This number is presented as the code to the word in the book – the participant is asked to look up at page 10, line 8 and word 9.

When explaining the mathematics of this trick the challenge is to prove that starting with any 3 digit number $\overline{xyz}$ (the first digit must be different from the last one: $x \neq z$), reversing its order, subtracting the smallest from the largest ($x > z$), reversing the order of the sum digits and adding the sum with its reversed version always gives 1089 as the result, irrespectively of the initial choice. This algebraic proof can be surprisingly simplified using the usual subtraction and addition algorithms (in Portugal subtraction is usually performed using the equal additions algorithm as below). This trick explanation is a good example to work algebra in middle school classes, providing students with a simple proof and showing them that variables do not appear only in equations and can be used in other contexts.

We start with the subtraction: since $x > z$ the units digit of the minuend has to be increased ($z + 10$) and the tens digit of the subtrahend is increased accordingly ($y + 1$) so that the difference is kept unchanged. Now the tens digit of the minuend (y) is again smaller than that of the subtrahend ($y + 1$) and the same procedure is applied to the tens and hundreds (underlined grey text).

Hundreds	Tens	Units
x	y +10	$z + 10$
$-\ z + 1$	$y + 1$	x
$x - z - 1$	9	$z + 10 - x$

Next we use the addition algorithm after reversing the order of the digits of the difference. For the units we have $(z + 10 - x) + (x - z - 1) = 9$ and for the hundreds we have $(x - z - 1) + (z + 10 - x) + 1 = 10$. The final result is always 1089.

	Hundreds	Tens	Units
	$x - z - 1$	9	$z + 10 - x$
+	$z + 10 - x$	9	$x - z - 1$
	10 ← (1)	**8**	**9**

Crystal Ball – Everybody in the classroom is asked to think of a two digit number, sum its digits and subtract the sum from the initial number. Then, by looking up the result in a projected table of numbers and symbols (see Figure 2), the magician/teacher is able to guess which symbol is attached to everybody's results. This trick is based on the flash applet "The Flash Mind Reader" [9] and is based on the divisibility by nine. The proof of the result is a simple example of algebraic reasoning: starting with a two digit number, $\overline{xy} = 10x + y$, and subtracting the sum of the digits always gives a multiple of nine: $(10x + y) - (x + y) = 10x + y - x - y = 9x$.

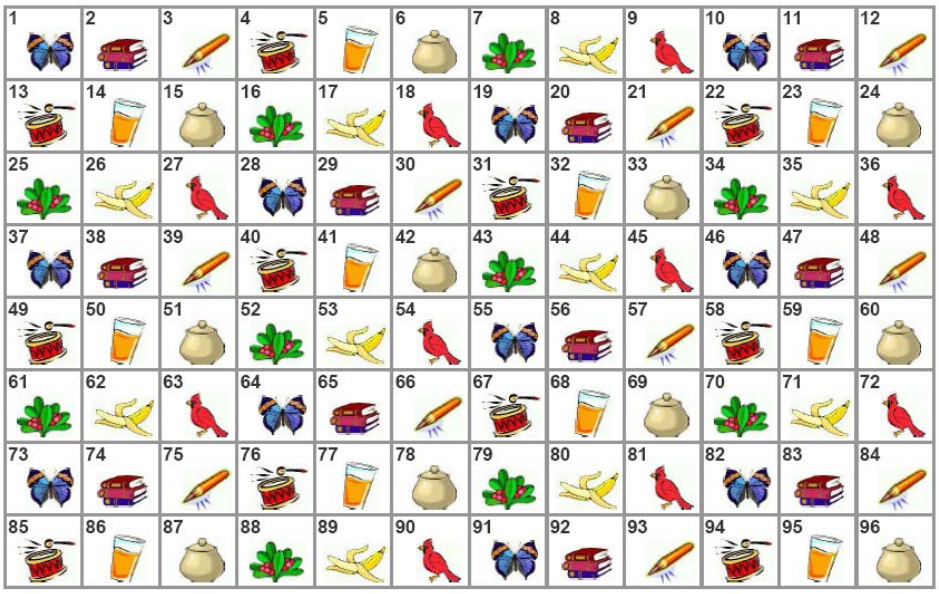

Figure 1. Example of a table for Crystal ball.

3 RESULTS

In this section we report a preliminary analysis of the questionnaire delivered towards the end of the term. The questionnaire was filled in by forty two students, corresponding to the number of students that attended the course regularly.

Students were asked (on a five point Likert scale: 1- not at all, 5- very much) if they enjoyed the performance of mathematical magic trick in the classes. The vast majority of the students gave a positive reply: 17 (42.9%) replied 4 - “I liked” and 19 (52.4%) replied 5 - “very much”. Only two (4.7%) students replied 3 - “indifferent”, and none gave a negative answer (2- “not much” or 1 – not at all”).

Students were also asked to rate on a five point Likert scale their agreement regarding several statements.

One of the statements was “The use of mathematical magic in the classroom contributes to a more positive attitude towards mathematics”. Once more, the vast majority of the students gave a positive reply: 85.7% of the students gave a positive answer, (full results can be seen in Table 1).

Another of these statements was “The use of mathematical magic tricks in math classes makes mathematics more boring and demotivating”. None of the students agreed with the statement as can been seen in Table 1.

Concerning the statement “The use of mathematical magic tricks in math classes makes the learning process more challenging, interesting and stimulating”, 80.0% of the students gave a positive answer, 20% replied 3 – neutral-, and none gave a negative answer.

Regarding the statement “The use of mathematical magic tricks in math classes contributes to better understand the importance of Mathematics”, 62.5% agree with it, 35.0% are neutral and only one respondent partially disagrees with it (no one is in total disagreement).

With regard to the statement “The use of mathematical magic tricks in math classes provides a less rigid method of learning, making it more interactive and interesting”, a large majority of students gave a positive reply (78.9% agrees with it), 18.4% are neutral and only one student partially disagrees.

Another statement was “The use of mathematical magic tricks in math classes serves only to entertain for a while”. Almost every one (88%) disagrees with it and only one student claims to partially agree with the statement.

Table 1. Distribution (absolute frequency; mean and St. Dev.) of the answers on the agreement with some given statements

Statement: The use of mathematical magic tricks in math classes…	1	2	3	4	5	NOp.	mean	s.d.
contributes to a more positive attitude towards mathematics.	0	0	6	17	19	0	4.3	0.7
makes mathematics more boring and demotivating.	39	2	2	0	0	0	1.1	0.5
makes the learning process more challenging, interesting and stimulating	0	0	8	17	15	2	4.2	1.1
contributes to better understand the importance of Mathematics	0	1	14	16	9	2	3.8	1.1
provides a less rigid method of learning, making it more interactive and interesting	0	1	7	16	14	4	4.1	1.4
serves only to entertain for a while	25	12	4	1	0	0	1.5	0.8

(1 – total disagreement; 3 – neutral; 5 – total agreement; NOp. – No answer)

When questioned if "the use of mathematical tricks in the classes is relevant?", 98% (almost all of the respondents – 41 out of 42 students) said "Yes" and only one student said "No". When asked to specify, amongst a given list, the reasons for this relevance, the most selected ones were:

- The classes are more interesting – 70.7% of the respondents selected this option.
- It stimulates the like for mathematics – 58.5% of the respondents selected this option.

As for "other reasons" specified by the respondents (open answers) we outline one which stated "It provides pre-service teachers with other teaching methodologies".

Students were asked to further specify the importance of using mathematical magic tricks in the classes, by rating it on a 5 point Lickert scale (1 – not important; 5 – very important). Ten students (23.8%) replied 5; 19 (45.2%) rated it as 4; 13 (31.0%) as 3; none replied 1 or 2. Although the mode is 4 and not 5, all students used rates 3 to 5. The overall average rate is 3.9 which is clearly positive, though not very close to the maximum. The use of mathematical magic tricks in the classroom is a complementary tool and not the primary methodology for teaching mathematics. It is therefore expected that students don't rate it as a very important resource.

When asked if the use of magic tricks in math classes helped them to enjoy mathematics more, the great majority of the students – 66.7% - answered "Yes" (33.3% answered "No"). Note that several students have a natural like for mathematics since they are taking a pre-service teacher degree for teaching elementary school, which includes mathematics teaching. It is therefore natural that some of the students will not increase their like for mathematics, which is already high.

4 CONCLUSIONS

As Hall and Pais [7] say, finding ways to engage students in the classroom and motivate them for learning mathematics is a challenge that most mathematics teachers face today. All contributions towards this goal are welcome and should be disseminated across these professionals. There is no single solution to this challenge and as in many other complex problems, several solutions must be combined and adapted to each context.

In this paper the authors propose one particular way to contribute to this goal: using mathematical magic tricks in the classroom. The case study here described, continues the preliminary work described in [7]. As in [7], the analysis of the field notes and the students' portfolios allows us to conclude that:

- In all classes there were always several students very interested in volunteering for the performance of the mathematical magic tricks, which always involved volunteers. This reveals that the students liked the activity and liked to participate in it.
- Quite often, several students were very surprised and intrigued with the magic trick and tried to figure out what was behind it. Sometimes they explicitly asked the teacher not to reveal the secret immediately so they could think on it. At other times the curiosity was so strong that they

asked the teacher to explain it immediately. Either way the students were curious and were developing their mathematical reasoning and engaging with mathematics.

- The topics taught which were subject to mathematical magic gained more meaning after performing the tricks because they had concrete examples of their application, with an extremely attractive purpose.
- Whenever the teacher forgot or intentionally began the lesson with other things than the magic trick, some students inquired if the teacher had forgotten the trick for this class.

The analysis of the collected information, through the questionnaire applied in the end of the term, the field notes and the students' portfolios, allows us to conclude that the use of mathematical magic tricks in the classroom, with the aim of engage students and motivate them to learn mathematics, has proved effective in math courses for pre-service teachers, namely CM I within the undergraduate degree LEB in a Portuguese university. Based on the opinion of the students involved in the study, we can conclude that, the use of mathematical magic in classes made the learning of mathematics more interesting, stimulating and challenging. It further contributed to a more positive view of Mathematics, and to better understand the importance of Mathematics. Nevertheless, this resource, in spite of being useful and welcome by the students, was not considered as an extremely important methodology (average rate 3.9 out of 5) corroborating the idea that no single methodology provides a full answer to the teaching challenges and that the use of mathematical magic tricks is mostly a complementary tool with high potential to increase the students' motivation towards mathematics.

As a final remark and reinforcing the preliminary conclusions obtained by Hall and Pais [7], we believe that the use of mathematical magic tricks on a regular basis throughout math courses is an added value in the process of learning with positive effects on both the motivation for learning and the learning itself.

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